Syllabus for the First Degree Programme in Mathematics
of the University of Kerala

Semester I
Methods of Mathematics

Code: MM 1141 Instructional hours per week: 4
No.of credits: 4

Overview of the course:

Mathematics education starts with natural numbers and in this part of the program, we take a closer look at their algebraic properties. Traditionally such a study comes under Theory of Numbers. Apart from introducing the basic facts about integers, the course aims to intro-duce the ideas of axioms and theorems and provide practice in writing proofs. Some glimpses into the applications of number theory to cryptography are also intended. In the ensuing semesters, the courses grow into ring theory and into an introduction to abstract algebra. The present course is based on the first five chapters of the text.

The present course is intended to present some of the methods of mathematics as used in number theory and Calculus. A study of the foundations of mathematics will be undertaken in the second semester, where some of the concepts will be revisited and put on a firm basis. Geometry forms the second major component of math education in schools. In the High School, students have seen how geometry is merged with algebra to form what is known as Analytic Geometry. Also, some problems in analytical geometry, such as finding the slope of a tangent and the area under a curve motivated much of calculus. So, we bring together these two branches of mathematics in this part of the course. It grows into real analysis in the ensuing semesters. The course is based on Chapters 1–4 and also Sections 4 and 5 of Chapter 11 of the text.

Module 1: Methods of Algebra

We attempt at a quick review of various sets of numbers, then mention the notion of partitions and equivalence relations, with which the students have had an acquaintance in their Higher Secondary class. Next, the Principle of Mathematical Induction and the Well-ordering principle are also touched upon. All these concepts find a detailed discussion in the foundation course in the ensuing semester. The use of the Well-ordering Principle in the definition of special types of numbers is to be emphasized and illustrated through the proof of the existence of lcm, as in Proposition 3.

Before introducing the Division Theorem, as in Section D, the usual process of long division to get the quotient and remainder must be recalled through examples. After proving the Division Theorem and the Uniqueness Proposition as in this section, the representation of natural numbers in different bases can be explained as in Section E. The last section of Chapter 2 on operations in different bases (Section F) need not be discussed.

The idea of gcd, studied in elementary class, is to be recalled next and the existence of such a number justified, as in Section A of Chapter 3. The idea of coprimality is also to be discussed here. Some of the important properties of coprime numbers, as in Exercises E9, E10 and E11 must be discussed.

Next, Euclid's Algorithm and some of its applications are to be discussed. After discussing the theoretical consequences of Euclid's Algorithm, namely Bezout's Identity and its corollaries, as in Section C, its practical use in solving indeterminate equations of the first degree is to be discussed, as in the text. The last two sections of this chapter on the efficiency of Euclid's Algorithm (Section D) and on incommensurability (Section E) need not be discussed. (See also http://en.wikipedia.org/wiki/Diophantine_equation)

Then, a discussion on primes and The Fundamental Theorem on Arithmetic, as given in the first three section of Chapter 4 is to be done. The last section on primes in an interval need not be discussed. (See, for example, http://en.wikipedia.org/wiki/Algorithm)

Finally, we introduce the new idea of congruences, as in Chapter 5. The fact that when an integer is divided by another, the dividend is congruent to the remainder modulo the divisor should be emphasized. In discussing the basic properties of congruence as in Section B, the fact that cancellation of common factors does not hold in general for congruences must be emphasized and illustrated through examples. Many of the tricks in Section C maybe familiar to the students and it must be emphasized that we are providing proofs here. Further properties of congruences as in Section D comes next and after that the solution of linear congruences as in Section E.

Text: Lindsay N. Childs, A Concrete Introduction to Higher Algebra. Second Edition, Springer

Module 2: Methods of Calculus-I

We start with a review of how the graph of an equation can be plotted, illustrated with examples, and move on to a working definition of a function. It must be emphasized through illustrations that not all equations connecting two variables give one variable as a function of the other, as in Example 1 of Section 1.2 of the text. (The notion of explicit and implicit definitions of functions and their graphs, as given in the first two parts of Section 3.6 can be discussed here itself.) Functions defined piecewise and their graphs must be specially mentioned and illustrated. Approximate solutions to problems through graphical methods are to be explained as in Example 7 of the section. Section 1.3 on using computers may be skipped, but the use of computers in plotting graphs should be demonstrated, using Open Source Software such as the plotting software gnuplot or the cas maxima. (See also http://www-groups.dcs.st-and.ac.uk/~history/Curves/Curves.html)

Some of the ideas in Section 1.4, such as arithmetic operations on functions, may be familiar to the students, but they should be reviewed. Other ideas such as symmetry, stretching and compression and translation may be new and should be emphasized. The same goes for Sections 1.5 and 1.6, with familiar ideas reinforced through illustrations and new ideas, especially physical applications, discussed in detail. Section 1.7 on mathematical modelling need not be discussed. But parametric equations, especially that of the cycloid, must be discussed in detail, as in Section 1.8.

Limits and continuity are concepts introduced (somewhat vaguely) in Higher Secondary class. In this course, these ideas are to be reinforced through graphs. They are to be made rigorous in the ensuing semester. Section 2.4 of Chapter 2 of the text is to be done only in semester II, but mention may be made of limits as $x \to \infty$, as well as infinite limits, which will be required in subsequent sections.

Module 3: Methods of Calculus-II

The notion of differentiation is also familiar to the students. Here, this idea is to be re-introduced through applications as in the first two sections of Chapter 3. Much of the material in Sections 3.3–3.7 maybe already seen, but they should be reviewed, emphasizing the graphical meaning and applications. The idea of implicit differentiation should be made clear, as in Section 3.6. The last section on approximations (Section 3.8) need not be discussed. (See also http://en.wikipedia.org/wiki/History_of_calculus)

Chapter 4 is also to be discussed in the same spirit, reviewing familiar concepts, explaining new concepts in detail and always emphasizing geometry and physical applications.

Module 4: Analytic Geometry

A detailed discussion on the equations of conic sections, as in Sections 11.4 and 11.5 of Chapter 11 is also part of this course. In Section 11.4, we begin with a demonstartion of conic sections as intersections of a plane with a double-napped cone. Then we move on to the equations of the conics in standard form, followed by a technique for sketching them. A method of finding asymptotes is discussed. This is followed by the section on translated conics and reflection properties of conic sections.

In Section 11.5, we discuss the equations of conics that are ‘tilted’ relative to the coordinate axes. This leads to a study of rotations of coordinate axes.

(See also http://en.wikipedia.org/wiki/Conic_sections and http://en.wikipedia.org/wiki/Dandelin_spheres)

Text: Howard Anton, et al, Calculus. Seventh Edition, John Wiley

References:

1. James Stewart, Essential Calculus, Thompson Publications, 2007.
2. Thomas and Finney,Calculus and Analytic Geometry, Ninth Edition, Addison-Wesley.
3. S.Lang, A first Calculus, Springer.

Distribution of instructional hours:

Module 1: 24 hours; Module 2: 18 hours; Module 3: 18 hours, Module 4: 12 hours