

## **Semester – 4**

**PC 16**

**MT04C16**

### **SPECTRAL THEORY**

**Text Book: Erwin Kreyszig, Introductory Functional Analysis with applications,  
John Wiley and sons, New York**

#### **Module I**

Strong and weak convergence, convergence of sequence of operators and functionals, open mapping theorem, closed linear operators, closed graph theorem, Banach fixed point theorem

(Chapter 4 - Sections 4.8, 4.9, 4.12 & 4.13 - Chapter 5 – Section 5.1 of the text)  
(25 hours)

## Module 2

Spectral theory in finite dimensional normed space, basic concepts, spectral properties of bounded linear operators, further properties of resolvent and spectrum, use of complex analysis in spectral theory, Banach algebras, further properties of Banach algebras.

(Chapter 7 - Sections 7.1. to 7.7 of the text)

(25 hours)

## Module 3

Compact linear operators on normed spaces, further properties of compact linear operators, spectral properties of compact linear operators on normed spaces, further spectral properties of compact linear operators, unbounded linear operators and their Hilbert adjoint operators, symmetric and self adjoint linear operators

(Chapter 8 - Sections 8.1 to 8.4 - Chapter 10 Sections 10.1 & 10.2 of the text)

(20 hours)

## Module 4

Spectral properties of bounded self adjoint linear operators, further spectral properties of bounded self adjoint linear operators, positive operators, projection operators, further properties of projections

(Chapter 9 - Sections 9.1, 9.2, 9.3, 9.5, 9.6 of the text)

(20 hours)

### Question Paper Pattern

	Part A Short questions	Part B Short essays	Part C Long essays
Module I	2	3	1
Module II	2	3	1
Module III	2	1	2
Module IV	2	1	2
Total	8	8	6

### References:-

1. Simmons, G.F, Introduction to Topology and Modern Analysis, McGraw –Hill, New York 1963.
2. Siddiqi, A.H, Functional Analysis with Applications, Tata McGraw –Hill, New Delhi : 1989
3. Somasundaram. D, Functional Analysis, S.Viswanathan Pvt Ltd, Madras, 1994
4. Vasistha, A.R and Sharma I.N, Functional analysis, Krishnan Prakasan Media (P) Ltd, Meerut: 1995-96
5. M. Thamban Nair, Functional Analysis, A First Course, Prentice – Hall of India Pvt. Ltd, . 2008

## ELECTIVE COURSES

PE 1

MT04E01

### ANALYTIC NUMBER THEORY

**Text:** Tom M Apostol, *Introduction to Analytic Number Theory*, Springer International Student Edition, Narosa Publishing House

#### Module 1 Arithmetic Functions Dirichlet Multiplication and Averages of Arithmetical functions

Introduction to Chapter1 of the text, the Mobius function  $\mu(n)$  the Euler totient function  $\varphi(n)$ , a relation connecting  $\mu(n)$  and  $\varphi(n)$ , the Dirichlet product of arithmetical functions, Dirichlet inverses and Mobius inversion formula, the Mangoldt function  $\Lambda(n)$ , multiplicative e functions and Dirichlet multiplication, the inverse of completely multiplicative functions, the Liouville's function  $\lambda(n)$ , the divisor function  $\sigma_\alpha(n)$ , generalized convolutions, formal power series, the Bell

series of an arithmetical function, Bell series and Dirichlet multiplication.

Introduction to Chapter2 of the text, the big oh notation, asymptotic equality of functions, Euler's summation formula, some elementary asymptotic formulas, the average order of  $d(n)$ , The average order of the divisor function  $\sigma_\alpha(n)$ , average order

of  $\varphi(n)$ , an application of distribution of lattice points visible from the origin, average order of  $\mu(n)$  and  $\Lambda(n)$ , the partial sums of a Dirichlet product, application to  $\mu(n)$  and  $\Lambda(n)$ .

(Chapter 2 sections 2.1 to 2.17 and Chapter 3 sections 3.1 to 3.11 of the text)

(30 hours)

#### Module 2 Some Elementary Theorems on the Distribution of Prime Numbers

Introduction to Chapter4, Chebyshev's functions  $\psi(x)$  and  $\mathcal{J}(x)$ , relation connecting  $\mathcal{J}(x)$  and  $\pi(x)$ , some equivalent forms of prime number theorem, inequalities of  $\pi(n)$  and  $p_n$  Shapiro's Tauberian theorem, applications of Shapiro's theorem, an asymptotic formula for the partial sum  $\sum_{p \leq x} \left(\frac{1}{p}\right)$ .

(Chapter 4 sections 4.1 to 4.8 of the text)

(15 hours)

#### Module 3 Congruences

Definition and basic properties of congruences, residue classes and complete residue systems, linear congruences, reduced residue systems and Euler – Fermat theorem, Polynomial congruences modulo  $p$ , Lagrange's theorem, applications of Lagrange's theorem, simultaneous linear congruences, the Chinese remainder theorem, applications of Chinese remainder theorem, polynomial congruences with prime power moduli

(Chapter 5 sections 5.1 to 5.9 of the text)

(30 hours)

#### Module 4 Primitive roots and partitions

The exponent of a number mod  $m$ . Primitive roots, Primitive roots and reduced systems, The non existence of Primitive roots mod  $2^\alpha$  for  $\alpha \geq 3$ , The existence of Primitive roots mod  $p$  for odd primes  $p$ , Primitive roots and quadratic residues. Partitions – Introduction, Geometric representation of partitions, Generating functions for partitions, Euler’s pentagonal-number theorem.

(Chapter 10 sections 10.1 to 10.5 &  
Chapter 14 sections 14.1 to 14.4 of the text) (15 hours)

#### Question Paper Pattern

	Part A Short questions	Part B Short essays	Part C Long essays
Module I	3	3	2
Module II	2	1	1
Module III	2	3	2
Module IV	1	1	1
Total	8	8	6

#### References:

1. Hardy G.H and Wright E.M , Introduction to the Theory of numbers, Oxford, 1981
2. Leveque W.J, Topics in Number Theory, Addison Wesley, 1961.
3. J.P Serre, A Course in Arithmetic, GTM Vol. 7, Springer-Verlag, 1973

PE 2

MT04E02

### COMBINATORICS

**Text Book:** Chen Chuan -Chong, Koh Khee Meng, Principles and Techniques in Combinatorics, World Scientific,1999.

#### Module I Permutations and Combinations

Two basic counting principles, Permutations, Circular permutations, Combinations, The injection and bijection principles, Arrangements and selection with repetitions ,Distribution problems

(Chapter I of the text) (20 hours)

#### Module II The Pigeonhole Principle and Ramsey Numbers

Introduction, The pigeonhole principle, More examples, Ramsey type problems and Ramsey numbers, Bounds for Ramsey numbers

(Chapter 3 of the text) (20 hours)

### Module III Principle of Inclusion and Exclusion

Introduction, The principle, A generalization, Integer solutions and shortest routes  
Surjective mappings and Sterling numbers of the second kind, Derangements and a  
generalization, The Sieve of Eratosathenes and Euler  $\varphi$ -function.  
(Chapter -4 Sections 4.1 to 4.7 of the text) (25 hours)

### Module IV Generating Functions

Ordinary generating functions, Some modelling problems, Partitions of integer,  
Exponential generating functions

#### Recurrence Relations

Introduction, Two examples, Linear homogeneous recurrence relations, General  
linear recurrence relations, Two applications  
(Chapter 5, 6 Sections 6.1 to 6.5) (25 hours)

#### Question Paper Pattern

	Part A	Part B	Part C	
	Short questions	Short essays	Long essays	
Module I	2	2	1	1
Module II	2	2	1	
Module III	2	2	1	1
Module IV	2	2	1	
Total	8	8	6	

--

#### References:-

1. V Krishnamoorthy, Combinatorics theory and applications, E. Hoewood, 1986
2. Hall,Jr, Combinatorial Theory, Wiley- Interscinice, 1998.
- 3.Brualdi, R A, Introductory Combinatorics, Prentice Hall,1992

PE 3

MT04E03

### CLASSICAL MECHANICS

Text: L. D. Landau and E. M. Lifshitz - MECHANICS, ( Third Edition )  
(Butter worth – Heinenann)

**Module 1:** Generalized coordinates, the Principle of least action, Galileo's relativity principle, the Lagrangian for a free particle, Lagrangian for a system of particle, energy, momentum, centre of mass, angular momentum, motion in one dimension, determination of the potential energy from the period of oscillation, the reduced mass, motion in a central field.

( Section 1 to 9, 11 to 14 of the text)

**Module 2:** Free oscillation in one dimension, angular velocity, the inertia tensor, angular momentum of a rigid body, the equation of motion of a rigid body, Eulerian angle, Euler's equation.

( Section 21, 31 to 36 of the text)

**Module 3:** The Hamilton's equation, the Routhian, Poisson brackets, the action as a function of the coordinates, Maupertui's principle.

( Section 40 to 44 of the text)

**Module 4:** The Canonical transformation, Liouville's theorem, the Hamiltonian – Jacobi equation, separation of the variables, adiabatic invariants, canonical Variables

( Section 45 – 50 of the text )

### References

1. M. G. Calkin, Lagrangian and Hamiltonian Mechanics, Allied
2. Herbert Goldstein, Classical mechanics, Narosa
3. K C Gupta, Classical mechanics of particles and Rigid Bodies, Wiley Eastern

PE 4

MT04E04

## PROBABILITY THEORY

**All questions shall be based on the relevant portions of the reference books given in the end of each module**

### Module - 1

Discrete Probability (Empirical, Classical and Axiomatic approaches), Independent events, Bayes theorem, Random variables, and distribution functions (univariate and multivariate), Expectation and moments, marginal and conditional distributions. Probability Inequalities (Chebychev, Markov). Modes of convergence, Weak and Strong laws of large numbers (Khinchine's Weak Law , Kolmogrov Strong Law, Bernaulli's Strong Law) Central Limit theorem (Lindeberg-Levy theorem ).

### References.

1. S.C. Gupta and V.K. Kapoor, Fundamentals of Mathematical Statistics, 11<sup>th</sup> Ed., Sultan Chand & Sons, 2011.
2. V.K. Rohatgi, An Introduction to Probability Theory and Mathematical Statistics, 2<sup>nd</sup> Ed. Wiley Eastern Ltd., 1986.

## **Module – 2**

Standard discrete and continuous univariate distributions (Binomial, Poisson, Negative binomial, Geometric, Exponential, Hypergeometric, Normal, Rectangular, Cauchy's, Gamma, Beta, ), Multivariate normal distribution, Wishart distribution and their properties.

### **References.**

**For univariate distributions, refer the book**

1. S.C. Gupta and V.K. Kapoor, Fundamentals of Mathematical Statistics, 11<sup>th</sup> Ed., Sultan Chand & Sons, 2011.

**For Multivariate distributions, refer the book**

2. T.W. Anderson, An Introduction to Multivariate Statistical Analysis, 3<sup>rd</sup> Ed., Wiley Interscience, 2003.

## **Module – 3**

Methods of estimation, properties of estimators, Cramer-Rao inequality, Fisher-Neyman criterion for sufficiency, Rao-Blackwell theorem, completeness, method of maximum likelihood, properties of maximum likelihood estimators, method of moments. Tests of hypothesis: most powerful and uniformly most powerful tests (Neyman – Pearson Lemma ).

### **References.**

**For Estimation, refer the book**

1. S.C. Gupta and V.K. Kapoor, Fundamentals of Mathematical Statistics, 11<sup>th</sup> Ed., Sultan Chand & Sons, 2011.

**For Tests of Hypothesis, refer the book**

2. V.K. Rohatgi, An Introduction to Probability Theory and Mathematical Statistics, 2<sup>nd</sup> Ed. Wiley Eastern Ltd., 1986.

## **Module- 4**

Gauss-Markov models, estimability of parameters, best linear unbiased estimators, Analysis of variance and covariance. One way and two way classification with one observation per cell.

### **References.**

1. D.D. Joshi, Linear Estimation and Design of Experiments, Wiley Eastern Ltd., 1990.
2. C.R. Rao, Linear Statistical Inference and its Applications, John Wiley, New York, 1965.
3. W.G. Cochran and G.M. Cox, Experimental Designs, 2<sup>nd</sup> Ed., John Wiley, New York, 1957.

## Question paper Pattern

	Part A Short questions	Part B Short essays	Part C Long essays
Module I	3	3	1
Module II	3	3	2
Module III	1	1	2
Module IV	1	1	1
Total	8	8	6

PE 5

MT04E05

### MATHEMATICAL ECONOMICS

**Text – 1:- Singh S.P, Anil K.Parashar, Singh H.P, Econometrics and Mathematical Economics, S. Chand & Company, 2002.**

**Text – 2:- JEAN E. WEBER, MATHEMATICAL ANALYSIS Business and Economic Applications, Fourth edition, HARPER & ROW PUBLISHERS, New York.**

**Module:-1 The theory of consumer behaviour-** Introductory, Maximization of utility, Indifference curve approach, Marginal rate of substitution, Consumer's equilibrium, Demand curve, Relative preference theory of demand, Numerical problems related to these theory part.

(Chaper – 13 .Sections 13.1, 13.2, 13.3, 13.4, 13.5, 13.6 & 13.13 of text - 1)(20 hours)

**Module:-2 The production function:-** Meaning and nature of production function, The law of variable proportion, Isoquants, Marginal technical rate of substitution, Producer's equilibrium, expansion path, The elasticity of substitution, Ridge lines and economic region of production, Euler's theorem, Cobb Douglas production function, The CES Production function, Numerical problems related to these theory parts.

(Chapter – 14. Sections 14.1, 14.2, 14.3, 14.4, 14.5, 14.6, 14.7, 14.8, 14.9, 14.10 & 14.11 of text - 1) (30 hours)

**Module:-3 Input – Output Analysis:-** Meaning of input – output, main features of analysis, Assumptions, Leontief's static and dynamic model, limitations, Importance and Applications of analysis, Numerical problems related to these theory parts..

(Chapter – 15. Sectios 15.1, 15.2, 15.3, 15.4, 15.5,15.6, 15.7, 15.8 & 15.9 of text - 1) (20 hours)



**Module:- 4 Difference equations** –Introduction, Definition and Classification of Difference equations, Linear Difference equations, Solution of Difference equations, Linear First-Order Difference equations with constant coefficients, Behaviour of the solution sequence, Equilibrium and Stability, Applications of Difference equations in Economic Models, The Harrod Model, The General Cobweb Model, Consumption Model, Income – Consumption – Investment Model.

( Chapter 6 Sections 6.1 to 6.5 of text 2)

(20 hours)

### Question paper pattern

	Part A	Part B	Part C
	Short questions	Short essays	Long essays
Module I	2	2	1
Module II	2	3	2
Module III	2	1	2
Module IV	2	2	1
Total	8	8	6

#### References:-

1. Allen.R.G..D, Mathematical Economics, 1959.
2. Alpha C Chiang, Fundamental methods of Mathematical Economics.
3. Koutsoyiannis. A, Modern Microeconomics, Macmillan.
4. Samuelson. P.A, Foundation of Economic Analysis.
5. Josef Hadar, Mathematical theory of economic behaviour, Addison-Wesley

PE 6

MT04E06

### COMPUTING FOR MATHEMATICS

#### Textbooks

**Text 1: E. Balagurusamy, Object Oriented Programming With C++, 4<sup>th</sup> Edition, Tata Mc Graw Hill, 2008.**

**Text 2: Leslie Lamport, LaTeX: A document preparation system, 2<sup>nd</sup> Edition, Addison-Wesley, 1994.**

**Module 1.** Principles of Object Oriented Programming, Beginning with C++, Tokens, expressions and control structures, Functions in C++.

- (Chapters 1-4 of text 1) (25 hours)
- Module 2.** Classes and Objects, Constructors and Destructors, Operator overloading and type conversions  
(Chapters 5-7 of text 1) (25 hours)
- Module 3.** Inheritance: Extending classes, Managing console I/O operation  
(Chapters 8 and 10 of text 1) (20 hours)
- Module 4.** Introduction to LaTeX: Getting started-Preparing an input file-The input Changing the type style-Symbols from other languages -Mathematical formulas Defining commands and environments. Other document classes-Books-Slides-Letter  
(Chapter 2,3,and 5 of Text 2) (20 hours)

**For this course a record book of the practical work is to be kept. A maximum of 3 weightage is to be awarded for the record and it is to be awarded by a committee of the HOD and the teacher in charge of the course. These 3 weightage is the weightages of the assignment, seminar and the internal viva.**

**If this paper is offered by the SDE or for private candidates the same is to be maintained and shall be produced before the viva board. The viva board can reserve a maximum of 10 marks for this record book.**

### Question paper Pattern

	Part A Short questions	Part B Short essays	Part C Long essays
Module I	2	2	2
Module II	2	2	2
Module III	2	2	1
Module IV	2	2	1
Total	8	8	6

### References

1. Stephen Prata, C++ Primer Plus, 5<sup>th</sup> Edition, Sams, 2004.
2. R. LaFore, Object Oriented Programming in C++, 4<sup>th</sup> Edition, Sams, 2011.
3. Deitel, and Deitel, C++ How to Program, 6<sup>th</sup> Edition, Prentice-Hall, 2008.
4. F. Mittelbach, M. Goossens The LaTeX Companion: 2<sup>nd</sup> Edition, *et.al.*, 2004.
5. Stroustup ,The C++ Programming Language, 3<sup>rd</sup> Edition, Addison-Wesley, 1997.

**Text -1- Ravindran. A, Don T Philips and James J Solberg., Operations Research Principle and Practice, 2<sup>nd</sup> edition, John Wiley and Sons.**

**Text – 2- Hamdy A. Thaha, Operations Research – An Introduction, 6<sup>th</sup> edition, Prentice Hall of India Pvt. Ltd.**

**Text – 3- K.V. Mital and C. Mohan, Optimization Methods in Operation Research and Systems Analysis, 3<sup>rd</sup> edition, New Age International Pvt. Ltd..**

**Text – 4 -Man Mohan, P.K. Gupta and Kanti Swarup, Operations Research, Sultan Chand and Sons.**

### **Module I: INVENTORY MODELS**

Introduction – Variables in an inventory problem – Objectives of inventory control – The classical E.O.Q. without shortages – The classical E.O.Q. with shortages – The Production Lot size (P.L.S) models – Nonzero Lead time – The Newsboy Problem (a single period model) – Lot size reorder point model – Variable lead times – The importance of selecting the right model.

(Chapter 8; Sections: 8.1 – 8.14 of text 1)

(20 hours)

### **Module 2: QUEUEING SYSTEMS**

Why study queues? – Elements of a queueing model – Role of exponential distribution (Derivation of exponential distribution; forgetfulness property) – Pure Birth and Death models – Relationship between the exponential and Poisson distributions – Generalized Queueing Models – Kendall notation – Poisson Queueing Models – Single server models and multiple server models – Machine servicing models – (M/M/R) : (GD/K/K) Model – (M/G/1) : (GD/) model – Pollaczek-Khintchine (P - K) formula.

(Chapter 17; Sections: 17.1 – 17.9 of text – 2)

(25 hours)

### **Module 3: DYNAMIC PROGRAMMING**

Introduction - Minimum path problem – Single additive constraint, additively separable return – Single multiplicative constraints, additively separable return - Single additive constraint, multiplicatively separable return – Computational economy in DP – Serial multistage models – Examples of failure – Decomposition – backward and forward recursions – Systems with more than one constraint – Applications of D.P to continuous systems.

(Chapter: 10; Sections: 10.1 – 10.12 of text – 3)

(20 hours)

### **Module 4: NETWORK SEQUENCING; SIMULATION MODELING**

Problem of sequencing – Basic assumptions – Processing n jobs through two machines – Optimum Sequence (Johnson Bellman) Algorithm - Processing n jobs through k machines – Processing of two jobs through k machines – Maintenance crew scheduling.

Simulation – Generation of random variables – Monte Carlo simulation – Sampling from probability distributions: 1. Inverse method, 2. Convolution method (&Box-Muller method), 3. Acceptance-Rejection method – Generic definition of events.

(Chapter: 12; Sections: 12.1 – 12.7 of text – 4)

Chapter: 18- Sections: 18.1 – 18.6 of text – 2)

(25 hours)

## Question Paper Pattern

	Part A Short questions	Part B Short essays	Part C Long essays
Module I	2	2	1
Module II	2	2	2
Module III	2	2	1
Module IV	2	2	2
Total	8	8	6

### References:-

1. Thomas L Satty, Elementary Queuing Theory, McGraw Hill Publishing Company.
2. Narasingh Deo, System Simulation with digital Computers, 7<sup>th</sup> edition, Prentice Hall India Pvt. Ltd., 1997.
3. Geoffrey Gordon, System Simulation, 2<sup>nd</sup> edition Prentice Hall India Pvt. Ltd, 1998.

PE 8

MT04E08

## SPECIAL FUNCTIONS

**Text Book:- Earl. D. Rainville, Special functions, Chelsa Publishing Company, New York, 1960.**

### Module – 1

Infinite products:- Introduction, definition of an infinite product, a necessary condition for convergence, the associated series of logarithms, absolute convergence, uniform convergence.

The Gamma and Beta functions:- The Euler and Mascheroni constant  $\gamma$ , the Gamma function, a series for  $\Gamma^{-1}(z) / \Gamma(z)$ , evaluation of  $\Gamma(1)$  and  $\Gamma^{-1}(1)$ , the Euler product for  $\Gamma(z)$ , the difference equation  $\Gamma(z + 1) = z\Gamma(z)$  the order symbols  $o$  and  $O$ , evaluation of certain infinite products, Euler's integral for  $\Gamma(z)$ , the Beta function, the value of  $\Gamma(z)\Gamma(1-z)$ , the factorial function, Legendre's duplication formulae, Gauss' multiplication theorem, a summation formula due to Euler, the behavior of  $\log \Gamma(z)$  for large  $|z|$

(Chapter 1 & 2 of text – Sections 1 to 22)

### Module – 2

The hypergeometric function:- The function  $F(a,b,c,z)$ , a simple integral form,  $F(a,b,c,1)$  as a function of the parameters, evaluation of  $F(a,b,c,1)$ , the contiguous function relations, the hypergeometric differential equation, logarithmic solution of the hypergeometric equation,  $F(a,b,c,z)$  as a function of its parameters, elementary series multiplications, simple transformations, relation between functions of  $z$  and  $1-z$ .

(Chapter 4 of the text – Sections 29 to 39)

**Module – 3**

Generalized Hypergeometric Functions: The function  ${}_pF_q$ , the exponential and binomial functions, a differential equation, other solutions of the differential equation, the contiguous function relations, a simple integral, the  ${}_pF_q$  with unit argument.

The Confluent Hypergeometric Functions: Basic properties of the  ${}_1F_1$ , Kummer’s first formula, Kummer’s second formula.

(Chapter 5 – Sections 44 to 50, Chapter 7 - Sections – 68, 69, 70)

**Module – 4**

Legendre Polynomials: A generating function, Differential recurrence relations, the pure recurrence relation, Legendre’s differential equation, the Rodrigue’s formula, Bateman’s generating function, additional generating functions, Hypergeometric forms of  $p_n(x)$ , Brafman’s generating function, special properties of  $p_n(x)$ .

Hermite Polynomials: Definition of  $H_n(x)$ , recurrence relations, the Rodrigue’s formula, other generating functions, integrals.

**Question paper pattern**

	Part A Short questions	Part B Short essays	Part C Long essays	
Module I	2	2	1	1
Module II	2	2	1	
Module III	2	2	1	1
Module IV	2	2	1	
Total	8	8	6	

**References:-**

1. M.A. Pathan, V.B.L.Chaurasia, P.K.Banerji, M.C.Goyal ,Special Functions and Calculus of Variations, Ramesh Book Depot, New Delhi, 2007.
2. Z.X. Wang, D.R. Guo, Special Functions, World Scientific Publishing Company, London, 1989.
3. N.M. Temme, Special Functions – An Introduction to the Classical Functions of Mathematical Physics, John Wiley & Sons, New York, 1996.
4. A.M. Mathai, H.J. Haubold, Special Functions for Applied Scientist, Springer, New York, 2008.

5. G.E. Andrews, R. Askey, R. Roy, Special Functions, Encyclopedia of Mathematics and its Applications 71, Cambridge University Press, Cambridge.1999.

PE 9

MT04E09

### THEORY OF WAVELETS

**Text Book:- Michael W. Frazier, An introduction to Wavelets through Linear Algebra, Springer- verlag, 2000.**

**Pre-requisites:-** Linear Algebra, Discrete Fourier Transforms, Elementary Hilbert Space theorem. ( No questions shall be asked from these sections.)

**Module – 1:-** Construction of Wavelets on  $\mathbf{Z}_N$ : The First Stage.  
(Chapter – 3 Section 3.1 of the text) (20 hours)

**Module – 2:-**Construction of Wavelets on  $\mathbf{Z}_N$  : The Iteration Step, Examples – Haar, Shannon and Daubechies).  
(Chapter – 3 Section 3.2 & 3.3 of the text) (20 hours)

**Module – 3:-**  $l^2(\mathbf{Z})$ , Complete Orthonormal sets in Hilbert Spaces,  $L^2[-\pi, \pi]$  and Fourier Series.  
(Chapter – 4 Section 4.1, 4.2 & 4.3 of the text) (20 hours)

**Module – 4:-** The Fourier Transform and Convolution on  $l^2(\mathbf{Z})$ , First-stage Wavelets on  $\mathbf{Z}$ , The Iteration step for Wave lets on  $\mathbf{Z}$ , Examples- Haar and Daubechies.  
(Chapter – 4 Section 4.4, 4.5, 4.6 & 4.7 of the text) (30 hours)

#### Question paper Pattern

	Part A Short questions	Part B Short essays	Part C Long essays	
Module I	2	2	1	1
Module II	2	2	1	
Module III	2	2	1	
Module IV	2	2	1	
Total	8	8	6	

#### References:-

1. Mayer, Wavelets and Operators, Cambridge University Press, 1993.
2. Chui, An Introduction to Wavelets, Academic Press, Boston, 1992.

PE 10

MT04E10

## SIGNAL THEORY

**Text Book:- Athanasios Papoulis and S. Unnikrishna Pillai, Probability, Random Variables and Stochastic Processes, Fourth edition, Tata McGraw-Hill, New Delhi.**

**Module – 1 General Concepts:**

Definitions, Systems with Stochastic Inputs, The Power Spectrum, Discrete-Time Processes, Simple problems  
(Chapter – 9, Sections 9.1 to 9.4 of the text) (22 hours)

**Module – 2 Random Walks and Other Applications.**

Random Walks, Poisson points and Shot Noise, Modulation.  
(Chapter –10, Sections 10.1 to 10.3 of the text) (22 hours)

**Module – 3 Spectral Representation**

Factorizations and Innovations, Finite-Order Systems and State Variables, Fourier Series and Karhunen-Loeve Expansions, Spectral representation of Random Processes, Simple problems.  
(Chapter – 11, Sections 11.1 to 11.4 of the text) (24 hours)

**Module – 4 Entropy**

Introduction, Basic Concepts, Coding, Channel Capacity, Simple Problems.  
(Proof of the channel Capacity theorem excluded)  
(Chapter – 14, Sections 14.1, 14.2, 14.5 & 14.6 of the text) (22 hours)

### Question paper Pattern

	Part A Short questions	Part B Short essays	Part C Long essays	
Module I	2	2	1	1
Module II	2	2	1	
Module III	2	2	1	1
Module IV	2	2	1	
<b>Total</b>	<b>8</b>	<b>8</b>	<b>6</b>	

**References:-**

1. Meldhi.J, Stochastic Process, Wiley Eastern, 1987
2. Srinivasan. C.K, Stochastic Processes, 2<sup>nd</sup> edition, Tata McGraw-Hill.
3. Karlin and Taylor, A First Course in Stochastic Processes.
4. Karlin and Taylor, A Second Course in Stochastic Processes.

**Text Book :- Gregor Kemper, A Course in Commutative Algebra, Springer, ISSN0072-5285, ISBN978-3-642-03544-6**

**Module :- 1 The Algebra-Geometry Lexicon – Hilbert’s Nullstellensatz**  
 Maximal ideals, Jacobson Rings, Coordinate Rings, Simple problems.  
 (Chapter1 Sections 1.1, 1.2 & 1.3 of the text) (25 hours)

**Module: -2 Noetherian and Artinian Rings.**  
 The Noether and Artin Properties for Rings and Modules, Noetherian Rings and Modules, Simple problems  
 (Chapter2 Sections 2.1 & 2.2, of the text) (20 hours)

**Module: - 3 The Zariski Topology**  
 Affine Varieties, Spectra, Noetherian and Irreducible Spaces, Simple problems.  
 (Chapter3 Sections 3.1, 3.2 & 3.3 of the text) (25 hours)

**Module: -4 A Summary of the Lexicon**  
 True Geometry: Affine Varieties, Abstract Geometry : Spectra , Simple problems  
 (Chapter4 Sections 4.1 & 4.2, of the text). (20 hours)

**Question paper Pattern**

	Part A Short questions	Part B Short essays	Part C Long essays	
Module I	2	2	1	1
Module II	2	2	1	
Module III	2	2	1	1
Module IV	2	2	1	
Total	8	8	6	

**References: -**

1. William W. Adams, Phillippe Lousstanaun, An Introduction to Grobnerbases, Graduate Studies in Mathematics 3, American Mathematical Society, 1994, [117]
2. Michael F Atiyah, Ian Grant Macdonald, Introduction to Commutative Algebra, Addison- Wesley, Reading, 1969[174]
3. Nicolas Bourbaki, General Topology, Chapters – 1 – 4, Springer, Berlin, 1993, [117, 118, 161].



**Text Book:** Mathai A.M., Saxena R.K., H.J. Houbold, The H-Function: Theory and Applications, Springer, 2010.

**Module-1**

**Introduction**

**A Brief Historical Background**

**Fractional Integrals:** Riemann-Liouville Fractional Integrals, Basic properties of Fractional Integrals, Illustrative Examples.

**Riemann-Liouville Fractional Derivatives,** Illustrative Examples.

(3.1, 3.2, 3.3 -3.3.1, 3.3.2, 3.3.3 - 3.4 - 3.4.1 of the text)

**Module-2**

**The Weyl Integral:** Basic properties of Weyl Integrals, Illustrative examples.

**Laplace Transform:** Laplace Transform of Fractional Integrals, Laplace Transform of Fractional Derivatives, Laplace Transform of Caputo Derivative.

(3.5 - 3.5.1, 3.5.2, 3.6 - 3.6.1, 3.6.2, 3.6.3 of the text)

**Module-3**

**Mellin Transforms:** Mellin Transform of the  $n^{\text{th}}$  Derivative, Illustrative Examples

**Kober Operators:** Erdelyi-Kober Operators

**Generalized Kober Operators**

(3.7 - 3.7.1, 3.7.2, 3.8 - 3.8.1, 3.9 of the text)

**Module-4**

**Saigo Operators:** Relations among the Operators, Power Function Formulae, Mellin Transform of Saigo Operators, Representation of Saigo Operators.

(3.10 - 3.10.1, 3.10.2, 3.10.3, 3.10.4 of the text)

**Question paper pattern**

	Part A Short questions	Part B Short essays	Part C Long essays
Module I	2	3	2
Module II	2	2	2
Module III	2	1	1
Module IV	2	2	1
Total	8	8	6

**References:-**

- (1) Dold.A, Eckmann. B, Fractional Calculus and its Applications, Springer Verlag, 1975.
- (2) Miller.K.S, Rose.B, An Introduction to Fractional Calculus and Fractional Differential Equation, John Wiley and Sons, 1993.
- (3) Nishimoto k, Fractional Calculus Integration and Differentiation of arbitrary order, Descartes press, Koriyama, 1991.

- (4) Oldham K.B, Spanier .J, Fractional Calculus theory and Applications of Differentiation and Integration to arbitrary order, Academic press, 1974.
- (5) Ian N Sneddon, The use of operators of Fractional Integration in Applied Mathematics,(Applied mechanic series), Polish Scientific publishers, 1979.
- (6) Lecture notes on Multivariable and Matrix variable calculus and Applications, Stochastic models, Edited by A.M. Mathai, Publication number – 40, SERC School notes, CMS, pala, Kerala.(phone- 04822-216317)

**PE 13**

**MT04E13**

## ALGORITHMIC GRAPH THEORY

**Text Book:- Gray Chartrand and O.R Oellermann , Applied and Algorithmic Graph**

**Theory, Tata McGraw- Hill Companies Inc**

**Module 1 : Introduction to Graphs and Algorithms**

What is graph? The degree of a vertex. isomorphic graphs. subgraphs, degree sequences. connected graphs. cutvertices and blocks. special graphs. digraphs. algorithmic complexity. Search algorithms, sorting algorithms. greedy algorithms., representing graphs in a computer.

( Capter 1 Sections 1.1 to 1.9, Chapter 2 Sections 2.1, 2.2 , 2.3, 2.5 and 2.6 of the text) (24 hours)

**Module 2: Trees, paths and distances**

Properties of trees, rooted trees. Depth-first search: a tool for finding blocks,. breadth – first search, . the minimum spanning tree problem

Distance in a graphs, distance in weighted graphs, .the centre and median of a graph. activity digraphs and critical paths.

(Chapter 3 sections 3.1 to 3.6 , Chapter 4 sections 4.1 to 4.4 of the text) (22 hours)

**Module 3: Networks**

An introduction to networks. the max-flow min-cut theorem. the max-flow min-cut algorithm . connectivity and edge connectivity . Mengers theorem.

( Chapter 5 sections 5.1 , 5.2 , 5.3 , 5.5 and 5.6 of the text ) (22 hours)

**Module 4 : Matchings and Factorizations**

An introduction to matchings . maximum matchings in a bipartite graph,. Factorizations. Block Designs.

(Chapter 6 sections 6.1 , 6.2 , 6.4 and 6.5 of the text) (22 hours)

### Question paper pattern

	Part A	Part B	Part C	
	Short questions	Short essays	Long essays	
Module I	2	2	1	1
Module II	2	2	1	

Module III	2	2	1	1
Module IV	2	2	1	
Total	8	8	6	

**Reference:-**

1. Alan Gibbons, Algorithmic Graph Theory, Cambridge University Press, 1985
2. Mchugh. J.A, Algorithmic Graph Theory, Prentice-Hall, 1990
3. Golumbic. M, Algorithmic Graph Theory and Perfect Graphs, Academic press.

**PE 14**

**MT04E14**

**CODING THEORY**

**Text :- Vera Pless 3<sup>rd</sup> Edition , Introduction to the theory of error coding codes, Wiley Inter Science**

**Module:-1** Introduction Basic Definitions Weight, Maximum Likelihood decoding Synarome decoding, Perfect Codes, Hamming codes, Sphere packing bound, more general facts.

(chapter 1 & Chapter 2 Sections 2.1, 2.2, 2.3 of the text) (25 hours)

**Module:-2** Self dual codes, The Golay codes, A double error correction BCH code and a field of 16 elements.

(Chapter 2 Section 2.4 & Chapter 3 of the text) (20 hours)

**Module:- 3** Finite fields

(Chapter 4 of the text) (20 hours)

**Module:- 4** Cyclic Codes, BCH codes)

(Chapter 5 & Chapter 7 of the text) (25 hours)

**Question paper Pattern**

	Part A Short questions	Part B Short essays	Part C Long essays
Module I	2	2	2
Module II	2	2	1
Module III	2	2	1
Module IV	2	2	2
Total	8	8	6

**References:-**

1. R-Lidi, G.Pliz, Applied Abstract Algebra, Springer Verlag.
2. J.H.Van Lint, Introduction to Coding Theory, Springer Verlag
3. R.E.Blahut, Error- Control Codes, Addison Wesley.

PE 15

MT04E15

**COMPLEX ALGEBRAIC CURVES**

**Text:- Frances Kirwan, Complex Algebraic Curves, London Mathematical Society Student Texts 23, Cambridge University Press, Cambridge.**

**Module:- 1** Introduction and background - Relationship with other parts of Mathematics – Number theory, Singularities and the theory of knots, Complex analysis, Abelian Integrals – Real Algebraic Curves – Hilbert’s Nullstellensatz, Techniques for drawing real algebraic curves, Real algebraic curves inside complex real algebraic curves, Important examples of real algebraic curves.  
(Chapter 1 of the text) (25 hours)

**Module:- 2** Foundations - Complex real algebraic curves in  $C^2$ , Complex projective spaces, Complex projective curves in  $P_2$ , Affine and Projective curves, Exercises (Simple problems.).  
(Chapter 2 of the text) (20 hours)

**Module:- 3** Algebraic Properties – Bezout’s theorem, Points of inflection and cubic curves, Exercises(simple problems)  
(Chapter 3 of the text) (25 hours)

**Module:- 4** Topological Properties –The degree – genes formula, Branched curves of  $P_1$ , Proof of degree-genus formula, Exercises (Simple problems)  
(Chapter 4 of the text – 4.1.1 & 4.1.2 excluded) (20 hours)

**Question paper Pattern**

	Part A Short questions	Part B Short essays	Part C Long essays	
Module I	2	2	1	1
Module II	2	2	1	
Module III	2	2	1	1
Module IV	2	2	1	
Total	8	8	6	

- References:-**
1. E. Arbarello, M. Cornalba, P. Griffiths and J. Harris, Topics in the theory of algebraic curves, Springer-Verlag(1985)
  2. E. Brieskorn and H. Knorrer, Plane Algebraic curves, Birkhauser-Verlag(1986)
  3. C.H. Clemens, A scrapbook of Complex curve theory, plenum(1980)
  4. J.L. Coolidge, A treatise on algebraic plane curves, Dover(1959)

**PE 16**

**MT04E16**

## **ALGEBRAIC GEOMETRY**

**Text:- Brendan Hassett, Introduction to Algebraic Geometry, Cambridge University Press, 2007.**

### **Module:- 1 Guiding problems**

Implicitization, Ideal membership, Interpolation

#### **Division algorithm and Grobner bases**

3rd chain conditions, Buchberger's Criterion.

(Chapter 1 – Sections 1.1 to 1.3, Chapter – 2 Sections 2.1 to 2.5) (30 hours)

### **Module:- 2 Affine varieties**

Ideals and varieties, Closed sets and the Zariski topology, Coordinate rings and morphisms, Rational maps, Resolving rational maps, Rational and unirational varieties.

(Chapter – 3 Sections 3.1 to 3.6) (22 hours)

### **Module:- 3 Elimination**

Projections and graphs, Images of rational maps, Secant varieties, joins, and scrolls.

#### **Resultants**

Common roots of univariate polynomials, The resultant as a function of the roots, Resultants and elimination theory.

(Chapter – 4 Sections 4.1 to 4.3 Chapter – 5 Sections 5.1 to 5.3)

(23 hours)

### **Module:- 4 Irreducible varieties**

Existence of the decomposition, Irreducibility and domains, Dominant morphisms.

#### **Nullstellensatz**

Statement of the Nullstellensatz, Classification of maximal ideals, Transcendence bases, Integral elements.

**Primary decomposition**

Irreducible

ideals, Quotient ideals, Primary ideals.

(Chapter:- 6 Sections 6.1 to 6.3) Chapter – 7 Sections 7.1 to 7.4 Chapter –8 Sections 8.1 to 8.3) (15 hours)

**Question paper Pattern**

	Part A Short questions	Part B Short essays	Part C Long essays	
Module I	3	3	1	1
Module II	2	2	1	
Module III	2	2	1	1
Module IV	1	1	1	
Total	8	8	6	

**References:-**

1. William Fulton, Algebraic Curves: An Introduction to Algebraic Geometry, Advanced Book Program, Redwood City, CA: Addison-Wesley, 1989.
2. Phillip Griffiths and Joseph Harris, Principles of Algebraic Geometry, New York: Wiley-Interscience, 1978.
3. Joe Harris, Algebraic Geometry, Graduate Texts in Mathematics, 133. New York: Springer-Verlag, 1992.

PE 17

MT04E17

**FRACTAL GEOMETRY**

**Text:- Kenneth Falconer, FRACTAL GEOMETRY Mathematical Foundations and Applications, John Wiley & Sons, New York.**

**Pre-requisites** – Mathematical background – A quick revision  
(Chapter 1 of the text).

No questions shall be asked from this section. (5 hours)

**Module:- 1 Hausdorff measure and dimension**

Hausdorff measure, Hausdorff dimension, Calculation of Hausdorff dimension-Simple examples, Equivalent definitions of Hausdorff dimension, Finer definitions of dimension.

**Alternative definitions of dimension**

Box counting dimension, Properties and problems of box counting dimension, Modified box counting dimension, Packing measures and dimension.

(Chapter 2 , 3 Sections 3.1 to 3.4 of the text.) (30 hours)

**Module: 2 Techniques for calculating dimensions**

Basic methods, Subsets of finite measure, Potential theoretic methods, Fourier transform methods.

**Local structure of fractals**

Densities, Structure of 1-sets, Tangents to s-sets.

(Chapter 4 & 5 of the text.) (25 hours)

**Module:- 3 Projections of fractals**

Projections of arbitrary sets, Projections of s-sets of integral dimension,

**Products of fractals** – Product formulae

(Chapter 6 & 7 of the text) (18 hours)

**Module:- 4 Intersections of fractals**

Intersection formulae for fractals, Sets with large intersection.

(chapter 8 of the text) (12 hours)

**Question paper Pattern**

	Part A Short questions	Part B Short essays	Part C Long essays
Module I	3	3	2
Module II	2	2	2
Module III	2	2	1
Module IV	1	1	1
Total	8	8	6

**Reference:-**

1. Falconer K.J, The Geometry of Fractal sets, Cambridge University Press, Cambridge.
2. Barnsley M.F, (1988), Fractals every where, Academic press, Orlando, FL.
3. Mandelbrot B.B, (1982), The Fractal Geometry of Nature, Freeman, San Francisco.
4. Peitgen H.O and Richter P.H, (1986), The Beauty of Fractals, Springer, Berlin.
5. Tamas Vicsek, Fractal Growth Phenomena, Second edition, World Scientific.

## LIE ALGEBRAS

**Text:- James E. Humphreys, Introduction to Lie Algebras and Representation Theory, Springer**

**Module:- 1 Basic Concepts**

Definition and first examples, Ideals and homomorphisms, Solvable and nilpotent Lie Algebras.

(Chapter I Sections 1, 2, & 3 of the text) (25 hours)

**Module:- 2 Semi simple Lie Algebras**

Theorems of Lie and Cartan, Killing form, Complete reducibility of representations.

(Chapter II Sections 4, 5, & 6 of the text) (20 hours)

**Module:- 3 Root Systems**

Axiomatics, Simple roots and Weyl group, Classification.(proof of Classification theorem excluded)

(Chapter III Sections 9, 10 & 11 of the text) (25 hours)

**Module:- 4 Isomorphism and Conjugacy Theorems**

Isomorphism theorem, Cartan Algebras, Conjugacy theorems

(Chapter IV Sections 14, 15, & 16 – 16.1 to 16.3 of the text) (20 hours)

### Question paper Pattern

	Part A	Part B	Part C	
	Short questions	Short essays	Long essays	
Module I	2	2	1	1
Module II	3	2	1	
Module III	2	3	1	1
Module IV	1	1	1	
Total	8	8	6	

**References:-**

1. J.G.F. Belinfante and B. Kolman, A survey of Lie Groups and Lie Algebras with computational methods and Applications, Philadelphia : SIAM, 1972.
2. N. Jacobson, Lie Algebras, New York – London, Wiley interscience, 1962.



3. H. Samuelson, Notes on Lie Algebras, Van Nostrand Reinhold Mathematical studies No. 23, New York: Van Nostrand Reinhold, 1969.

PE 19

MT04E19

## ALGEBRAIC TOPOLOGY

**Text :-** Fred H. Croom - Basic concepts of Algebraic Topology (Springer verlag) ISBN 0-387-90288-0 New York and ISBN 3-540-90288-0 Berlin . Heidelberg

Chapters 1-5 (All sections and Theorems)

### Module 1

Geometric complexes and Polyhedra-Introduction-Examples-Orientations of geometric complexes-Chains-Cycles-boundaries and Homology groups-Examples of Homology groups-The structure of Homology groups-The Euler-Poincare Theorem-Pseudomanifolds and the Homology groups of  $S^n$ .

### Module 2

Simplicial approximations-Induced homomorphisms on the Homology groups-The Brouwer fixed point Theorem and related results.

### Module 3

The Fundamental group-The covering homotopy property for  $S$ -Examples of fundamental groups-the relation between  $H_1(K)$  and  $\pi_1(K)$ .

### Module 4

Covering spaces -Definition and some examples-Basic properties of covering spaces-Classification of covering spaces-Universal covering spaces.

### Question paper Pattern

	Part A Short questions	Part B Short essays	Part C Long essays	
Module I	2	2	1	1
Module II	2	2	1	

Module III	2	2	1	1
Module IV	2	2	1	
Total	8	8	6	

### References

1. B.K.Lahiri-A first Course in Algebraic Topology (Second Edition)-Narosa Publications-ISBN 81-7319-635-4
2. Glen E.Bredon-Topology and Geometry (Springer)- ISBN 81-8128-266-3.
3. Joseph J.Rotman-An Introduction to Algebraic Topology (Springer) –ISBN 81-8128-179-9.

**PE 20**

**MT04E20**

## **FINANCIAL MATHEMATICS**

**Text:- Robert J Elliott, P.Ekkehardkopp, Mathematics of Financial Markets, Second edition, Springer**

### **Module:-1 Pricing of Arbitrage**

Introduction: Pricing and Hedging, Single-Period Option Pricing Models, A General Single- Period Model, A Single- Period Binomial Model, Multi-Period Binomial Models, Bounds on Option Prices  
(Chapter:- 1 Section 1.1 to 1.6 of the text) (24 hours)

### **Module:- 2 Martingale Measures**

A General Discrete-Time Market Model, Trading Strategies, Martingales and Risk-Neutral Pricing, Arbitrage Pricing: Martingale Measures, Strategies Using Contingent Claims, Example: The Binomial Model, From CRR to Black-Scholes  
(Chapter:- 2 Section 2.1 to 2.7 of the text) (22 hours)

### **Module:-3 The First Fundamental Theorem**

The Separating Hyper Plane Theorem in  $\mathbf{R}^n$ , Construction of Martingale Measures, Path wise Description, Examples, General Discrete Models.  
(Chapter:- 3 Section 3.1 to 3.5 of the text) (22 hours)

### **Module:- 4 Complete Markets**

Completeness and Martingale Representation, Completeness for Finite Market Models, The CRR Model, The Splitting Index and Completeness, Incomplete Models: The Arbitrage Interval, Characterisation of Complete Models.  
(Chapter:- 4 Section 4.1 to 4.6 of the text) (22 hours)

## Question paper Pattern

	Part A	Part B	Part C	
	Short questions	Short essays	Long essays	
Module I	2	2	1	1
Module II	3	2	1	
Module III	2	3	1	1
Module IV	1	1	1	
Total	8	8	6	

### References:-

1. L.U. Dothan, Prices in Financial Markets, Oxford University Press, New York, 1990
2. D.Duffie, Future markets, Prentice-Hall, Englewood cliffs, N.J, 1989.